Math 253
Spring 2021

Name: Key

Quiz 5.1

Be sure to show sufficient work for each problem.

1. (5 points) Consider the sequence defined by $x_n = 4\left(\frac{1}{3}\right)^n - 2$ for $n \geqslant 0$. Convince yourself (but you don't have to write anything down for this part) that $\lim_{n\to\infty} x_n = -2$. Find an index n_0 so that for all $n \geqslant n_0$, $|x_n + 2| < 0.001$

Want: $|x_n + 2| < 0.001$

LHS: $|x_n + 2| = \left|4\left(\frac{1}{3}\right)^n\right| = \frac{4}{3^n}$

Want: $\frac{4}{3^n} < 0.001 \longleftrightarrow 4000 < 3^n \longleftrightarrow \log_3(4000) < n$

7.5

$n_0 = 8$

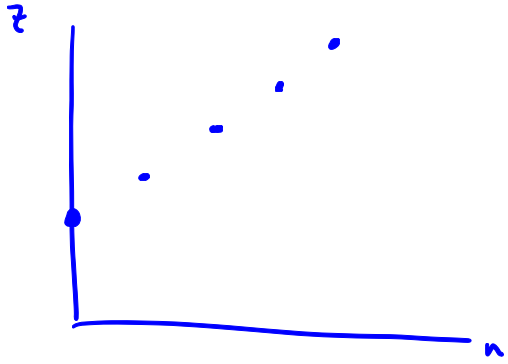
2. (2 points) Is the sequence defined by $y_0 = 3$ and $y_{n+1} = 2y_n - 1$ for $n \geqslant 0$ arithmetic, geometric, or neither?

$y_0 = 3,\ y_1 = 5,\ y_2 = 9$

Diff: 2 4 → not arithmetic

Ratio: $\frac{5}{3} \neq \frac{9}{5}$ → not geometric

3. (3 points) Is the sequence defined by $z_n = 2n + 3$ for $n \geqslant 0$ bounded from above, below, both, or neither? Is z_n monotone after $n_0 = 0$?



z_n is bounded below by 3 and is not bounded above

z_n is monotone (increasing) after 0.